Astronomy C Key

November 15-23, 2025

Instructions:

➔ You will have **50 minutes** to complete your exam. At the end of this time period, you must stop working. As per tournament rules, you may use resources in the form of a **physical binder OR files that are accessible offline on your team's device(s).**

➔ Significant figures will not be scored. Give **three (3)** decimal places for all calculated answers. Partial credit will be awarded, so attempt as many questions as you can!

➔ All multiple choice and true/false questions have **exactly one (1) correct answer** unless otherwise stated in the question. **Point values for questions are marked in parentheses after the question.**

➔ **Ties will be broken in accordance with the national Science Olympiad rules in this order:**
   ◆ **Section C (Astrophysics)**
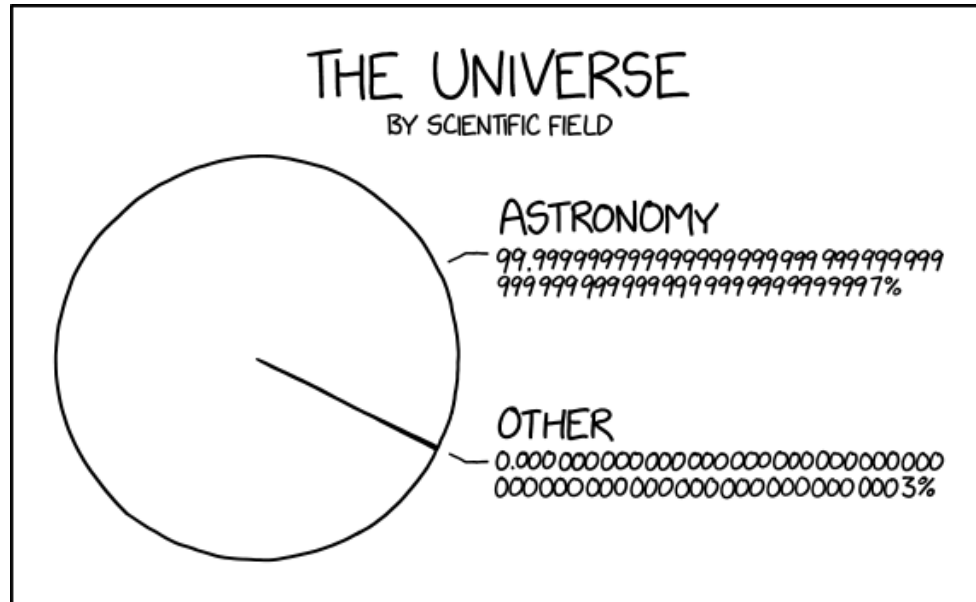   ◆ **Section B (DSO)**
   ◆ **Section A (General)**

School/Team Name:

_____

Team Number: ____

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Section A (General)

(45 Pts)

1) What are the two main elements that make up the interstellar medium? (Circle two answers) (1)

    A. Carbon

    B. Hydrogen

    C. Iron

    D. Helium

2) What explains the difference between the directions of the Henyey and Hayashi tracks on the H-R Diagram? (1)

    A. Hayashi protostars contract while staying the same in temperature, while Henyey protostars contract while also increasing in temperature, due to the larger size and greater amount of fuel Henyey protostars have

    B. Hayashi protostars contract while staying the same in temperature, while Henyey protostars contract while also increasing in temperature, due to the Henyey stars' radiative core which better retains the gravitational energy of contraction

    C. Hayashi protostars contract while increasing in temperature, while Henyey protostars contract while staying the same in temperature, due to the Henyey stars' radiative core which can radiate away excess heat

    D. Hayashi protostars contract while decreasing in temperature, while Henyey protostars increase in temperature because of a greater rate of gravitational contraction

3) Which of the following is an accurate statement regarding the evolution of a cool, red dwarf star of 0.5 solar masses? (1)

    A. It never develops a radiative core, instead generating energy through the proton-proton chain reaction, which is moved around through its convective zone.

    B. It develops a radiative core when it moves onto the main sequence, and will eventually transition to a neutron star, being below the Chandrasekhar limit of 1.44 solar masses.

    C. After leaving the main sequence, it undergoes a helium flash and transitions to the asymptotic giant phase.

    D. It develops a radiative core when it moves onto the main-sequence phase, and at that time, begins to generate energy through the proton-proton chain reaction.

4) What is the main factor responsible for how stars differ from each other in the way that they perform nuclear fusion (CNO cycle vs proton-proton chain)? (1)

    A. The core temperature determines the average kinetic energy of protons and helium nuclei. Higher temperatures increase the probability that nuclei can overcome the Coulomb barrier, allowing higher-threshold reactions such as the CNO cycle.

    B. The core temperature controls the energy generation rate needed to balance gravitational pressure. Low-mass stars only need the slower pp chain to maintain hydrostatic equilibrium, while high-mass stars require the faster CNO cycle, which is possible only at higher core temperatures.

    C. The core temperature determines the equilibrium concentrations of intermediate nuclei like carbon, nitrogen, and oxygen in the core. At high temperatures, these intermediates are produced rapidly enough to allow the CNO cycle to dominate.

    D. The difference in fusion pathways is determined by the star's surface abundance of carbon, nitrogen, and oxygen. Stars with more CNO elements at their surface can perform the CNO cycle.

5) Cores of stars on the asymptotic giant branch are _____ (1)

    A. Sustained through higher-level carbon and helium fusion, indicative of giant stars

    B. Made of carbon & oxygen mainly, supported by electron degeneracy pressure, and inert without fusion

    C. Being steadily heated up and expanded by the burning shells of helium, giving it the name "giant"

D. Are all above the Chandrasekhar limit of 1.44 masses and have an inert core, which enables them to collapse into a neutron star after the helium shells finish burning out

6) Why are faint blue galaxies significant to our understanding of star formation? (1)
A. Faint blue galaxies orbit around many bigger galaxies like the Milky Way.
B. Faint blue galaxies are modern galaxies that have an unusually rapid rate of blue star formation, due to an increased presence of dark matter which accumulates more stellar gas into the galaxy to form bigger stars.
C. Faint blue galaxies demonstrate an unusually low rate of star formation, making them "faint" or less luminous, and it is not yet completely determined why.
D. Billions of light years away, objects from billions of years ago are still visible to us. Faint blue galaxies are one of those archaic objects - small galaxies with a rapid rate of star formation, the building blocks of mature galaxies.

7) Why is a brown dwarf often called a "failed star"? (1)
A. They are stars which ignited nuclear fusion but did not have enough hydrogen to sustain for long, leading to them burning out quickly
B. It is a colloquial term, referring to the fact that brown dwarfs achieve a much lower level of brightness than regular stars.
C. They never gained enough mass for a significant amount of nuclear fusion in the first place
D. They are stars which experienced a collision with another star, extinguishing their light

8) What is it that prevents stellar cores below the Chandrasekhar mass limit, after the death of their star, from collapsing inward by their own self-gravity to form a white dwarf? (1)

A. The atoms repel and move away from each other due to the Pauli Exclusion Principle
B. The electrons in the atoms magnetically repel the electrons in other atoms, keeping the atoms apart.
C. The Coulomb repulsion between positively charged nuclei grows dominant at small separations, preventing the star from contracting any further.
D. The residual thermal motion of particles in the stellar core maintains a pressure that counteracts gravitational collapse, but also halts expansion and stops it from glowing.

9) When a stellar core exceeds the Chandrasekhar limit in mass, why does the process that would have prevented the core from collapsing (the correct answer from Question 8), not apply anymore? (1)

    A. Magnetic or Coulomb repulsion becomes negligible at the extreme densities that happen when a stellar core gets large.

    B. The residual thermal motion of particles in the stellar core stops under high gravitational pressure.

    C. If the stellar core is so heavy that the atoms would have to repel each other faster than the speed of light to counteract gravity, which is not possible, any repulsive force is overcome and it collapses.

    D. The magnetic attractive forces between the positively and negatively charged hydrogen ions in the core can override the repulsive force if the stellar core gets quite massive and gains a large number of ions, causing its collapse.

10) Assume that a white dwarf is in a close binary with a helium-rich donor star. If the mass transfer rate onto the white dwarf is high enough, stable helium burning can occur on its surface. Which of the following is a likely evolutionary consequence? (1)

    A. The donor star is slowly stripped of its helium, producing strong outflows of gas, that, over millions of years, can lead to the star's dissipation.

    B. The system can become an X-ray binary system, where the white dwarf emits strong X-rays due to it being powered by nuclear fusion.

    C. The system can become an X-ray binary system, but the white dwarf primarily emits soft X-rays due to its relatively thin layer of helium.

    D. The white dwarf will soon undergo a rapid and large outburst as mass accretes on it exponentially.

11) Why is it relatively easy to calculate the absolute magnitude of, and therefore the distance to, a Type 1a supernova? (1)

    A. They generally all have a similar absolute magnitude because all Type 1a supernovae are, by definition, triggered by similar conditions (white dwarfs exceeding Chandrasekhar limit)

    B. They periodically increase and decrease in brightness, and that period is correlated with their absolute magnitude.

    C. They generally all have a similar absolute magnitude because most Type 1a supernovae, due to gravitational pairing, have stellar partners of a similar mass.

D. Their absolute magnitude is correlated with the color of the supernova, as the color of the supernova is related to the wavelengths of light it produces and therefore its energy level.

12) Which of the following is one difficulty astronomers encounter in using Type 1a supernovae as distance indicators as compared to Cepheid variables? (1)

A. Type Ia supernovae are so bright that they frequently saturate telescope detectors, making accurate measurements impossible.
B. Because supernovae are powered by hydrogen fusion in their outer layers, their light curves vary wildly and cannot be standardized.
C. Type Ia supernovae occur mostly in young star-forming galaxies, making them less useful for measuring distances in old elliptical galaxies.
D. The explosions are rare and unpredictable, so astronomers cannot plan to observe them in advance.

13) Which of the following is one difficulty astronomers encounter in using Cepheid variables as distance indicators as compared to RR Lyrae variables? (1)

A. Cepheids are significantly more luminous, making them harder to detect in nearby galaxies without causing errors in ultrafine detectors.
B. Cepheids exhibit a period–luminosity relation that requires more careful calibration than for an RR Lyrae star, and requires corrections for metallicity effects.
C. Cepheids vary on much longer timescales than RR Lyrae stars, making it harder to measure their periods accurately.
D. Cepheids are generally found only in globular clusters, which complicates their use in external galaxies.

14) Which of the following is one difficulty astronomers encounter when using RR Lyrae variables as distance indicators as compared to Mira variables? (1)

A. RR Lyrae stars are much fainter than Mira variables, limiting their use to nearby galaxies.
B. RR Lyrae stars often suffer from heavy dust obscuration in the infrared, making their luminosity calibration uncertain.
C. RR Lyrae stars have highly irregular pulsation periods compared to Miras, requiring longer observations to determine their light curves.
D. RR Lyrae stars' periods are strongly dependent upon the amount of heavy elements found inside them, luminosity relation, complicating their calibration.

15) Which of the following is an accurate statement about the differences between the protostar stage of a variable star and that of a regular star? (1)

    A) Variable stars develop to become so due to luminosity fluctuations caused by unstable accretion from their circumstellar disks during their protostar stage, while regular protostars have relatively steadier luminosity evolution.

    B) Variable protostars reach the Hayashi track on the Hertzsprung–Russell diagram through a different path than regular protostars, bypassing the Kelvin–Helmholtz contraction phase.

    C) Variable protostars often show strong, irregular changes in brightness due to developing hydrogen fusion significantly earlier than regular stars; the overheat at the beginning of their life is what leads their luminosity to become unstable.

    D) The protostar stage of a variable star is not significantly different from that of a regular star, as protostars have not started nuclear fusion yet; a variable star only becomes variable later in its lifetime.

16) Which of the following statements is NOT true about the chemical composition of variable stars? (1)

    A) The pulsations of Cepheid variables are strongly influenced by the ionization states of helium in their envelopes, rather than by major changes in their overall elemental abundances.

    B) RR Lyrae stars, being older and metal-poor, show weaker metallic absorption lines than younger variables, but their pulsation mechanism still depends on helium ionization zones.

    C) Mira variables, especially carbon-rich types, show spectral features dominated by molecules like C_2, CN, and SiO, reflecting ongoing changes in nuclear fusion reactions in their outer atmospheres each cycle.

    D) The metallicity of Cepheid variables plays a role in calibrating their period–luminosity relation, which is why their chemical composition must be carefully considered when using them as distance indicators.

17) What is the role of the Kappa mechanism in regulating temperature in variable stars? (1)

    A. The Kappa mechanism, when it increases the opacity of the outer layer of the star, leads to heat not being able to escape from the stellar core, increasing its temperature and causing it to expand.

    B. The Kappa mechanism, when it increases the opacity of the star in general, can slow down hydrogen fusion, decreasing the temperature.

C. The Kappa mechanism causes the star to increase in opacity when the stellar core cools and expands, increasing temperature again and therefore stabilizing the star's temperature.

D. The Kappa mechanism causes the variable star to expand and contract significantly in size as the nuclear fusion rates periodically increase, causing stellar expansion, and then decrease as the gas density decreases.

18) Which method of measuring distance is NOT part of the "Cosmic Ladder?" (1)

A. Parallax
B. Measuring distance to standard candles
C. Redshift measurement
D. Astrometric Microlensing

19) Kepler's third law is $T^2 = 4\pi^2 r^3 / GM$. However, if you use astronomical units as measure of distance, years as measure of time, and solar masses (M) as a measure of mass, what does the equation simplify to, and why? Explain logically. (1)

In astronomical units, years, and solar masses, Kepler's third law becomes "the square of the period equals the cube of the distance." This works because the units are defined so that Earth's orbit is one astronomical unit in radius, one year in period, and the Sun's mass is one solar mass. Substituting these values makes the constant in the full equation equal to one, leaving only the simple relation. For stars with different masses, the period squared equals the distance cubed divided by the mass in solar units.

20) Planetary nebulas last for about… (1)

A. 20,000 years
B. 200,000 years
C. 2,000,000 years
D. 20,000,000 years

21) Why does a planetary nebula have "planet" in its name? (1)

A. If there are planets orbiting the star, they help to gravitationally hold the planetary nebula's gas in place after the star dies. Therefore, the nebula is called "planetary".
B. They are not actually related to planets, they just happen to look like them.
C. The name "planetary" comes from the gas in the nebula being made of planet-like solid chunks.

D. Planetary nebulas are named for planets because the ejected gas tends to form rings like planetary orbits, hence the appearance of circular structures in many of their images.

22) True or False: Herbig AE/BE stars have less developed magnetic fields than T-Tauri stars. (1)

23) True or False: The metallicity of a star has a nearly mathematically linear relationship with its age. (1)

**24)** The emission spectrum for a cold molecular cloud would be... (1)

A. Dominated by rotational transitions of molecules (especially CO) in the radio to millimeter range.
B. Bright hydrogen Balmer and Lyman series lines in the optical and UV, produced by collisional excitation of atomic hydrogen in the cold gas.
C. Strong synchrotron radiation in the X-ray band, generated by relativistic electrons spiraling along magnetic field lines in the molecular gas.
D. Thermal blackbody emission peaking in the visible range, with prominent spectral absorption lines of neutral hydrogen and helium.

Questions 25 and 26 will refer to the following graph, which represents a certain aspect of a star's properties:
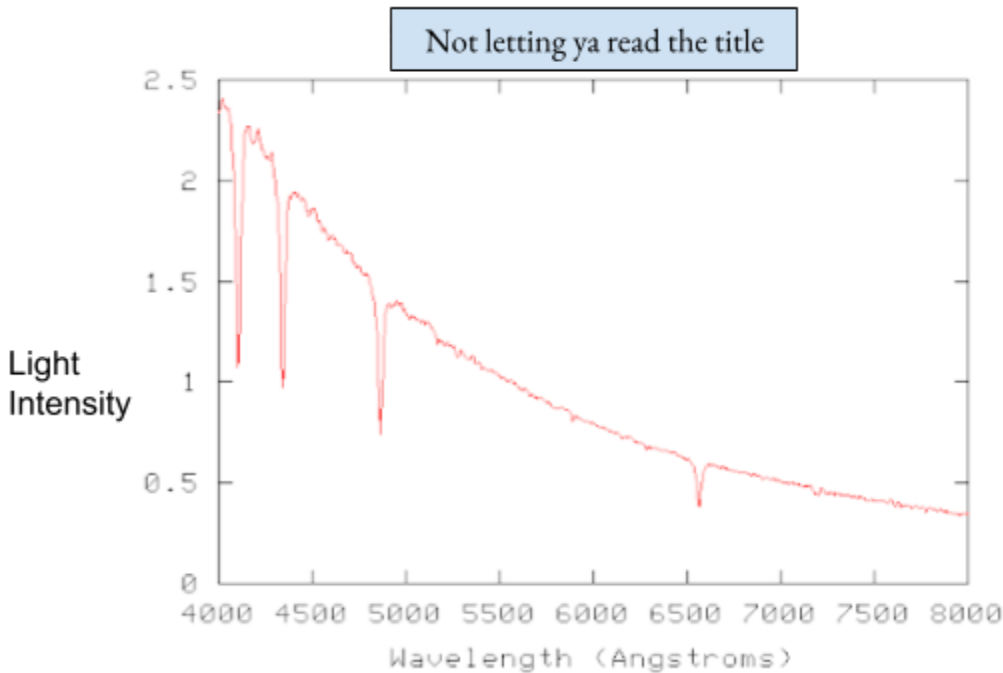
25) What color do you think the star is? (1)

    A.   Deep blue
    B.   Orangish-yellow
    C.   Deep red
    D.   Almost pure white

26) (FRQ) In a general sense, what is causing sudden dips in the light intensity at specific wavelengths? Hint…think about whether a star is similar to a blackbody. Is the star not emitting light at those places, or is it being absorbed by something? (2)

Specific elements in the star's atmosphere are absorbing light at specific wavelengths.

27) (FRQ) Suppose a certain star is classified as an A-type main-sequence star. Discuss how its color index (B–V) would reflect its blackbody properties, and why the measured spectrum of colors emitted may differ from the idealized blackbody prediction (at least two reasons). (5)

Accept any answer that:

    1)   Emphasizes that the star, due to being similar to a blackbody (an object that emits wavelengths of light as a function of temperature) will emit blue light due to its heat.
    2)   Gives two valid reasons for why the color index may differ from that if the star were a blackbody, i.e. why the light coming off the star may not be *fully* a function of the temperature alone.

28) (FRQ) Explain why a collapsing molecular cloud often fragments into multiple objects rather than forming a single star, and support your answer logically. Hint: this has to do with the existence of variations in density within regions of the cloud. (5)

A collapsing cloud fragments into multiple stars because initial turbulence and density variations create dense regions within the cloud that can collapse faster than the entire cloud.

Do not give full points for a vague answer (such as one that simply refers to multiple regions of density as a cause for multiple stars forming) or any other kind of answer that does not hit at the idea that denser regions within a molecular cloud collapse *faster* than the whole cloud.

29) (FRQ) What is the reason that the period and the luminosity of Cepheid variables are mathematically correlated? (4)

Accept any answer which explains that Cepheids with larger masses take longer to complete the Kappa Mechanism because it takes longer for the steps of the mechanism (opacity increase, stellar contraction, etc.) to happen when the star is big.

30) (FRQ) Why is it that (generally) the younger a star is, the more metallicity it has? (4)

When stars die, they expel metals into the universe, which accumulates in future star formation regions and causes future generations of stars to have these metals inside them; therefore, the younger the star is the more metallicity it has

Section B (DSOs)

(50 Pts)
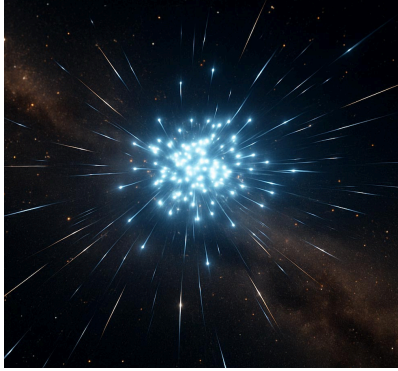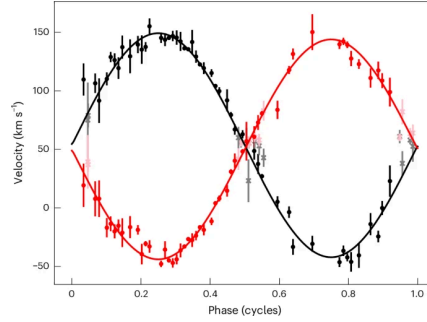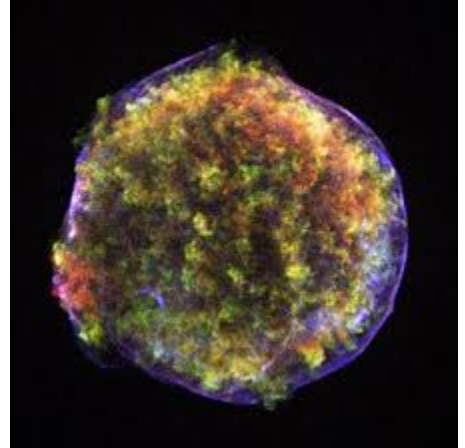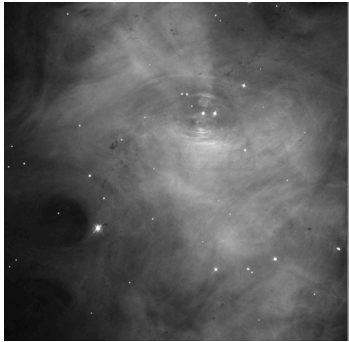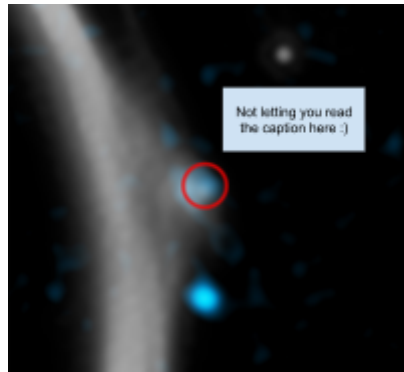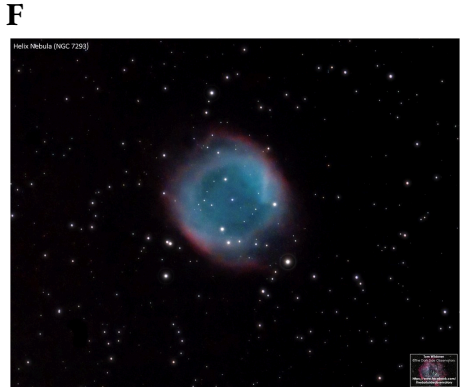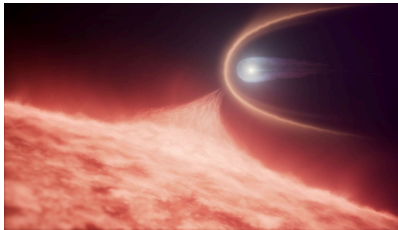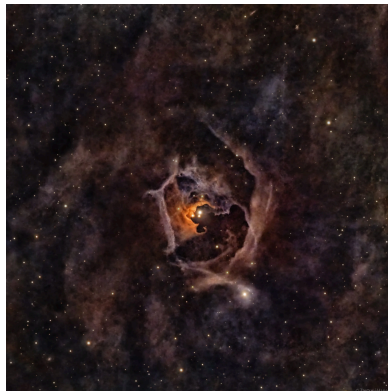
A	**B**	**C**
D	**E**	**F**
G	**H**	**I**

L

1) The questions below are about WDJ181058.67+311940.94. (5)

   a) What is the total mass of this system? (0.5)

   $1.56\ M_\odot$

   b) What is the temperature of the higher mass white dwarf? (0.5)

   ~17000 K - 18000 K

   c) If we assume that the primary white dwarf has a radius of 1 Earth Radii, what is the luminosity of the primary in solar luminosities? (0.5)

   $$\left(\frac{L}{L_\odot}\right) = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4 \Rightarrow 4\pi\sigma R^2 T^4 = 4\pi\sigma(6378e3\ m)^2(17260\ K)^4 \Rightarrow 0.0066\ L_\odot$$

   d) According to an article published in Nature Astronomy, what is the total explosion energy (in joules) of a Type Ia supernovae of this system? (1)

   10^{44}

   e) What are the typical surface gravity dex (decimal exponent) for white dwarfs? (1)

   8 - 9 dex

   f) What image depicts WDJ181058.67+311940.94? (0.5)

   Image B

   g) The projected absolute magnitude for this binary is -16, higher than the typical -19.3 for Type Ia Supernovae. Why is this the case? (1)

   Helium mass transfer from roche-lobe overflow causes a premature detonation before the white dwarves have merged. Also because both stars are not near the Chandresekar Mass, they have less Nickel in their interiors, which causes a subluminous supernova.

2) Which of the following is an accurate description of spectral/emission features observed in Sharpless 29? (1)

a) ==Sharpless 29 exhibits strong H-alpha emission (red) from hydrogen gas ionized by young massive stars, along with blue light from dust-scattered starlight, plus significant absorption in dark filaments.==

b) Sharpless 29's spectrum is dominated by broad molecular absorption bands in the ultraviolet, primarily from H_2O and CO_2 ices, with negligible hydrogen recombination lines.

c) Sharpless 29 emits intense synchrotron radio continuum across all radio bands due to relativistic electrons in its core, but shows almost no optical line emission.

d) Sharpless 29's visible spectrum shows strong neutral helium absorption lines matching those of early-type stars embedded in the nebula, while hydrogen emission is absent.

3) The questions below pertain to Image D (4)

     a) Identify the DSO shown in Image D (0.5)

        Crab Nebula

     b) What is Earth's distance from this DSO? (0.5)

        ~6200 - ~6500 ly

     c) What is the apparent magnitude of the object inside this DSO (0.5)

        ~16.65

     d) Photons with an energy of 100 TeV have been observed from this DSO. What portion of the Electromagnetic spectrum does this correspond to? (1)

        Gamma Rays

     e) What is the mass of the nebula surrounding the DSO? (0.5)

        $4.6 \pm 1.8\, M_{\odot}$

     f) What is part of the luminosity of the central object powered by? (1)

        The "spin-down" luminosity of the Crab Pulsar. Part of angular energy of the pulsar's spin is contributed to its luminosity

4) The following questions pertain to Image E (4.5)

     a) Identify the DSO shown in Image E. (Note: the part circled in red is not the DSO, it's just an object near the DSO). (0.5)

        The Bone or G359.13

     b) What object within this DSO (let's call this object X from here on) is circled in the image? (0.5)

        Pulsar G359 (0.5 points for just "pulsar" or just "G359")

     c) Explain briefly what happened when Object X crashed into the DSO and why. There are two key effects, one relating to its magnetics / electromagnetics and one about its structure. (2)

        The object's magnetic field interfered with the radio wave emission of the bone (1 pt) and caused a "fracture" in its structure (1 pt).

    d) (0.5) Which telescope observed that X had crashed into the DSO? Chandra

    e) (0.5) In what constellation is this DSO located? Sagittarius

    f) (0.5) True or False: this DSO is an active region of star formation. False

5) The following questions pertain to Image H. (6.5)

    a) Identify the DSO shown in Image H. (0.5)

       HP Tauri

    b) What is the other variable star designation of brightest star in this photo? (0.5)

       V1025 Tauri

    c) What type of nebulae is in the image? (0.5)

       Reflection Nebulae

    d) Some of the stars visible in the photograph are closer binaries that were visually observed despite being below optical limits of telescopes. How was this possible? (2)

       Lunar occultation data analysis that occulted a component star's light, revealing companion stars for fractions of a second.

    e) The age range of this DSO is 3 - 8 million years old. Assuming near solar analogs, is this too young or old for these stars to be fusing hydrogen? Why? (2)

       Too young. The Kelvin Helmholtz timescale for a solar mass star and radii is approximately 20 - 30 mya, longer than the maximum estimated age of HP Tauri

    f) What distance are the stars from the surrounding nebulae? (An order of magnitude will be accepted) (1)

       ~2000 - 5000 AU

6) The following questions pertain to Image L. (5)

    a) Identify the DSO shown in Image L. (0.5)

       Mira (Omicron Ceti)

    b) Historically, what is the significance of this DSO? (1)

       One of the first non-supernova variable stars known, along with Algol

    c) Is the secondary star shown in the photo gravitationally related to the primary? (2)

       No, that is not Mira B, it is a background star (HD 14411)

    d) What is the age of this DSO? (0.5)

       ~6 billion years

    e) What wavelength is this DSO brightest in? (0.5)

       Infrared

       What are the apparent magnitudes of this object? (0.5)

       2.0 - 10.1

7) The following questions pertain to Image F (4.5).
   a) Identify the DSO in Image F (0.5)

      Helix Nebula

   b) What are the two nicknames of this DSO? (1)

      Eye of God (0.5 pts) and Eye of Sauron (0.5 pts)

   c) A large body of matter once crashed into this DSO. How did the EM emission of the object at its center change as a result? (0.5)

      It began to emit x-rays.

   d) The high velocity wind from the central part of this DSO interacts with stable gas from the outer regions to produce what distinctive structures? (1)

      Cometary knots

   e) Is the temperature of the central part of this DSO in the tens of thousands of kelvin, hundreds of thousands, or millions? (0.5)

      Hundreds of thousands

   f) This DSO emits high quantities of molecular hydrogen. Why is this surprising to scientists? (1)

      The intense radiation from the white dwarf should destroy the hydrogen molecules.

8) The questions below are about the Ophion Star Family (6)
   a) What image depicts this DSO? (0.5)

      Image A

   b) Cosmic cavities around which star in Ophiuchius could have caused the high velocity dispersion of the Ophion Star Family? (1)

      Rho Ophiuchi

   c) What data release of Gaia was the Ophion Star Family found? (0.5)

      Gaia Data Release 3

   d) How would cosmic cavities in the interstellar medium cause stars to disperse rapidly? (2)

      "The rapid mass loss in the bubble means that the young stars in Ophion had less gravity restraining their dispersal."

   e) What age constraint did astrophysicists apply to discover Ophion? (1)

      <20 mya

   f) Roughly how many stars are in this family? (1)

      ~1000

9) The following questions pertain to Image C (4)
   a) Identify the DSO in image C. (0.5)

      Tycho's supernova, Tycho's SNR, or SN 1572

b) This DSO challenged the long held theory of the "unchangeable heavens". Why is this the case? (1)

When the star exploded as a supernova, it suddenly became visible in the sky, appearing as a "new star". This debunked the theory that the heavens would remain the same for all eternity and that nothing new would appear.

c) Is this object a standard candle? If so, why is its brightness inherently knowable? (1)

It is a type 1a supernova, so it was caused by a white dwarf accumulating enough material to reach the Chandrasekhar limit. Since the explosion happens immediately after the white dwarf reaches said mass, all type 1a supernovae have the same mass and therefore the same peak brightness.

d) What is the origin of this object's name, and why is its namesake significant to the field of astronomy? (0.5)

It is named after Tycho Brahe, an early modern astronomer who challenged traditional assumptions about the universe like geocentrism.

e) (0.5) What is the peak magnitude of this DSO? -4

f) (0.5) True or false: This DSO has been observed since BCE times. False

10) The following questions pertain to Image G. (5.5)

a) Identify the DSO in Image G. (0.5)

Janus or ZTF J203349.8+322901.1

b) **(TIEBREAKER) (3 pts)** This object's chemical composition differs on both its hemispheres, and there are multiple theories why. Which theory do you agree with the most? Name it, briefly explain why, and include counterarguments against other major theories. Grade this based on knowledge

c) True or false: most objects of this DSO's type contain heavier elements within their core and lighter elements in their atmosphere. (0.5)

d) What is the average surface temperature of this object in Kelvin? (Answers will be accepted within 5000 kelvin of the correct answer.) (0.5)

35000 kelvin

e) What is the distance from this object to Earth in light years? (Answers will be accepted within 100 light years of the correct answer.) (0.5)

1300 ly

f) Which university discovered this object? (0.5)

Caltech

11) The following questions pertain to Image I (4)

a) Identify the DSO in image I. (0.5)
   Orion, Orion Nebula, Orion Molecular Cloud, or Orion Molecular Cloud Complex, or M42

b) Explain why massive stars forming in or around this object can both trigger *and* inhibit further star formation in nearby regions (at least 1 way for each). (1)
   Example response: Massive stars can initiate the gravitational collapse of gas and dust, leading to the formation of a new generation of stars. However, the stellar wind from these stars can clear out the gas in the region, simultaneously inhibiting the stars' formation. Responses may differ from the key; accept any response that is accurate

c) Why do astronomers use radio or infrared telescopes instead of optical ones when studying this DSO? (0.5)
   Because these wavelengths can penetrate interstellar dust clouds and reveal cooler, hidden objects that are invisible in visible light.

d) Astronomers have detected carbon monoxide (CO) in this DSO using radio waves. This fact implies the presence of another molecule in this DSO - one that cannot be measured directly with radio. What molecule is this? (0.5)
   H_2 or hydrogen

e) Is this DSO tens, hundreds, thousands, or tens of thousands of light years in diameter? (0.5)
   Hundreds

f) What is the total mass of this DSO in solar masses? An order of magnitude will be accepted. (1)

   100,000 solar masses.

Section C (Astrophysics)

(50 Pts)

1) **Betelgeuse Binarity.** Because most stars are estimated to have some stellar multiplicity, Betelgeuse was long expected to have a stellar companion as the cause of semi-regular variability. In July 2025, the Gemini North Observatory made a possible direct detection of Betelgeuse's companion. The stellar companion will be referred to as *Siwarha*. (15 pts)
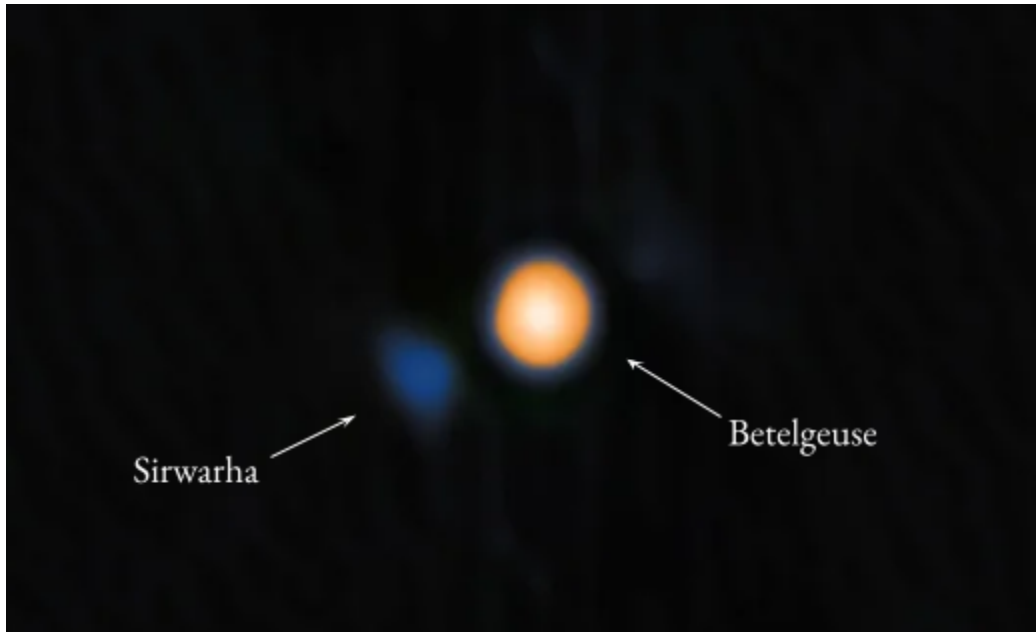
a) The method used to detect Betelgeuse's companion was speckle imaging. What is a basic overview of how this method works? (2)

Taking multiple extremely short exposure images and stacking them together to create high-subarcsecond resolution images - especially of close binary star systems

b) Sirwarha is assumed to be a main-sequence star with an estimated mass of $1.6\,M_\odot$. How many times brighter is Betelgeuse than Sirwarha? $(M_{Betelgeuse} = -6)$ (2)

$$\left(\frac{L}{L_\odot}\right) = \left(\frac{M}{M_\odot}\right)^{3.5}$$

$$L_{Sirwarha} = 1.6^{3.5} = 5.181\,L_\odot$$

$$M_{Sirwarha} = 4.83 - 2.5\log\left(5.181\frac{L}{L_\odot}\right) = 3.044$$

$$\left(\sqrt[5]{100}\right)^{3.044-(-6)} = 4145 \text{ times brighter}$$

c) With the Solar Constant to be $1400\ W/m^2$ and $m_{Sun} = -26.74$, what is the apparent magnitude of Sirwarha? Assume the (highly uncertain) distance to Betelgeuse is 490 ly. (4)

$$m_{Sirwarha} = -26.74 - 2.5log\left(\frac{\frac{5.181\ L_{\odot} \cdot 3.926 \cdot 10^{26}\ W/L_{\odot}}{4\pi(490\ ly \cdot 9.46 \cdot 10^{15}\ m/ly)}}{1400\ W/m^2}\right) = 8.94$$

d) Siwarha was located around 0.052" away from Betelgeuse. What is the corresponding distance of Siwarha from Betelgeuse in AU? Assume projected physical separation = true physical separation. (2)

$$0.052" = 206265\left(\frac{x\left(1.5 \cdot 10^{11}\ \frac{m}{AU}\right)}{490\ ly \cdot 9.46 \cdot 10^{15}\ \frac{m}{ly}}\right),\ x = 7.79\ AU$$

Prior to the direct imaging of Sirwarha in July 2025, a 2024 study found evidence that a not-yet directly observed companion could be a solution to Betelgeuse's 2,170-day secondary periodicity. Assume the orbital inclination of Sirwarha is 90 degrees.

e) Assuming the Betelgeuse-Sirwarha system to have a mass of $16.6\ M_\odot$, determine the semi-major axis of Sirwarha in AU. (2)

$$P^2 = \frac{a^3}{M_{total}},\ \left(\frac{2170\ days}{365\ days}\right)^2 = \frac{a^3}{16.6\ M_\odot}$$

$$a = 8.372\ AU$$

f) What is the orbital velocity of Sirwarha in km/s? (Relative to Betelgeuse) (3)

$$v = \sqrt{G\left(16.6\ M_\odot \cdot 1.989\ \cdot 10^{30}\ kg/M_\odot\right)\left(\frac{2}{7.79\ AU} - \frac{1}{8.372\ AU}\right)}\ \text{(Convert AU to meters)}$$

$$v = 44.907\ km/s$$

2) The Stellar Swindle. Prior to the hypothesis of nuclear fusion in 1920 by Arthur Eddington, the processes that powered the Sun had a wide range of theories, ranging from coal burning, energy from continuously-infalling meteors and comets, to gravitational contraction. However, none of these theories matched the timescales geologists known at the time, which predicted the Earth to be at least 2 billion years old. Physicists Lord Kelvin and Hermann Helmholtz set out to solve this discrepancy. (16 pts)

a) For a mass m falling onto the surface of the Sun (mass M) from a distance d, what is the resulting equation for the energy released through the change in gravitational potential energy? (2)

$$\Delta U = U_f - U_i \quad -\frac{GMm}{R_\odot} - \left(-\frac{GMm}{d}\right)$$

If it is assumed that $d \to \infty$ and the energy released by infalling matter to be equal to $E = -\Delta U$ with the efficiency coefficient ε, then the luminosity generated by infalling matter will be

$$L = \varepsilon\frac{GM\dot{m}}{R_\odot} \qquad\qquad \dot{m} = \frac{dm}{dt}$$

b) Assuming $\varepsilon = 0.1$, what is the rate of infalling matter that is required to generate the Sun's current luminosity in $M\oplus/yr$? (3)

$$3.9 \cdot 10^{26}\,W = (0.1)\frac{G\left(1.989\cdot10^{30}kg\right)\left(\frac{dm}{dt}\right)}{\left(6.96\cdot10^{8}m\right)}$$

$$\frac{dm}{dt} = 2.045 \cdot 10^{16}\,\frac{kg}{s} \to 0.108\,M\oplus/yr$$

c) How many years would this take for the Sun's mass to double? (4)

$$2.045 \cdot 10^{16}\,\frac{kg}{s} = 3.24 \cdot 10^{-7}\,M_\odot/yr$$

$$2\,M_\odot = 1\,M_\odot + \left(3.24 \cdot 10^{-7}\,M_\odot/yr\right)t$$

$$t = 3.086\ million\ years$$

Obviously, infalling matter is not what powers the Sun. However, Kelvin and Helmholtz set out for another type of gravitational energy source: gravitational contraction. The basic GPE equation, $U = -\frac{GMm}{r}$, is used for two point masses separated by a distance r. Obviously this equation would not work inside spherical masses. A revised version of the equation, shown below, will be used instead.

$$dU = -G\frac{m(r)dm}{r}$$

d) What does $m(r)$ represent? (1)

   The mass enclosed below radius r

e) What is dm the mass of? (1)

   The mass of a small spherical shell above $m(r)$

Through some generous assumptions, integration, differentiation, and the Virial Theorem, we get $E = |U| = \frac{3GM^2}{10R}$

f) At its current luminosity, how many years would the Sun last on gravitational contraction? (3)

$$t = \frac{3GM_\odot^2}{10R_\odot L_\odot} = 9.267 \; million \; years$$

g) What is this process of gravitational contraction called? (1)

   The Kelvin-Helmholtz Mechanism

h) What celestial objects utilize this as a primary power source? (1)

   Protostars/T-Tauri Stars

3) Pulsating Variable Star. Mira variable stars are known for their wide amplitudes in luminosity. The prototype star Mira (Omicron Ceti) is located in the constellation of Cetus. (19 Pts)

a) By what factor does Mira's luminosity <u>increase</u> from its minimum to its maximum if the apparent magnitude ranges from 3.5 - 9.0? (2)

$$\left(\sqrt[5]{100}\right)^{9.0-3.5} = 158.5$$

b) We will approximate Mira's luminosity function as a sinusoidal curve:

$$L_\odot(\phi) = a_L sin(2\pi\phi) + 9500 \ [L_\odot]$$

   i) What does the ϕ symbol represent? (0.5)

       Phase in a sinusoidal graph, 0 - 1 representing a full period

   ii) What does the a_L in the function represent? (0.5)

       Amplitude of the luminosity

   iii) What does the $9500 \ L_\odot$ represent? (0.5)

       Average luminosity during variability

c) Calculate the amplitude of Mira's luminosity in $L_\odot$. (3)

$$L_{avg} = \frac{L_{highest}+L_{lowest}}{2} = 9500 \ L_\odot, \quad \frac{L_{highest}}{L_{lowest}} = 158.5, \quad a_L = \frac{L_{highest}-L_{lowest}}{2}$$

$$L_{highest} + L_{lowest} = 19000 \ L_\odot, L_{highest} = 158.5 L_{lowest}$$

$$158.5 L_{lowest} + L_{lowest} = 19000 \ L_\odot$$

$$L_{lowest} = 119.122 \ L_\odot, \qquad L_{highest} = 18880.878 \ L_\odot$$

$$a_L = \frac{18880.878 \ L_\odot - 119.122 \ L_\odot}{2} = 9380.878 \ L_\odot$$

$$L_\odot(\phi) = \left(9380.878 \ L_\odot\right)sin(2\pi\phi) + 9500 \ [L_\odot]$$

(In Visible Light)

Now that we have a complete luminosity function $L_\odot(\phi)$, we can derive a radii function $R(\phi)$ based on observational quantities. $R_\odot(\phi)$ will be defined as below:

$$R_\odot(\phi) = a_R sin(2\pi[\phi - \varphi]) + R_0$$

d) Interferometry observations show that the radius of Mira pulsates from $332\ R_\odot - 402\ R_\odot$.

     i) Solve for R_0 (0.5)

$$R_0 = \frac{402\ R_\odot + 332\ R_\odot}{2} = 367\ R_\odot$$

     ii) What does the φ symbol represent? Why is it needed? (1)

The phase shift. We must shift our radius function since the hottest temperature in the variability occurs at the lowest radius. However, our radius function at phase 0 starts at the average radius R_0, so we must shift our graph (question below).

e) Based on the luminosity graph, what is the appropriate value of φ for $R_\odot(\phi)$? (0.5)

0.5

f) What is the function for radius $R_\odot(\phi)$? (1)

$$a_R = \frac{402\ R_\odot - 332\ R_\odot}{2} = 35\ R_\odot$$

$$R_\odot(\phi) = \left(35\ R_\odot\right)sin(2\pi[\phi - 0.5]) + 367\ R_\odot$$

g) Assume that the average luminosity (t = 0 days). What is the luminosity (in $L_\odot$) at t = 104

days? The approximate period of Omicron Ceti is 332 days. (1.5)

$$Phase\ \phi = \frac{104\ Days}{332\ Days} = 0.313$$

$$L_\odot(0.313) = \left(9380.878\ L_\odot\right)sin(2\pi(0.313)) + 9500\ [L_\odot]$$

$$= 18155.480\ L_\odot$$

h) Why can't an accurate $T_\odot(\phi)$ be created, knowing $L_\odot(\phi)$ and $R_\odot(\phi)$? Hint: (Wavelength band)

(1)

Our luminosity function $L_\odot(\phi)$ is in visible light only, we cannot accurately make

$T_\odot(\phi) = \left[\frac{L_\odot(\phi)}{[R_\odot(\phi)]^2}\right]^{1/4}$ **with a bolometric luminosity function**

i) What do you notice about the peaks and troughs of both graphs? (0.5)

The lowest radii coincides with the highest luminosity and vice versa.

j) What factor has the greatest impact on luminosity based on the graph? Why? (0.5)

Temperature; in $L = 4\pi\sigma R^2 T^4$, $L \propto R^2$ but $L \propto T^4$ is greater.

Asymptotic Giant Branch stars like Mira are known for their high velocity stellar winds and
extreme mass loss rates, up to $10^{-4}\ M_\odot/yr$.
Stellar companions could be accreting the ejected material onto a disk or onto their surfaces.
Mira B is a white dwarf star with a mass of 0.6 $M_\odot$

orbiting Mira A at an average distance of 100 AU.

k) What is the angular separation between Mira A and Mira B in arcseconds? Is this detectable
with ground-based observatories? (2)

$$206265\frac{100\ AU \cdot 1.5 \cdot 10^{11}[m/AU]}{300\ [ly] \bullet 9.46 \bullet 10^{15}[m/ly]} = 1.1", \text{Yes}$$

l) A study revealed that Mira B accretes as much as 1% of the material ejected by Mira A. Using the mass loss rate of 10^{-7} $M_\odot/yr$ for Mira A, what is the apparent magnitude of Mira B, assuming all material is accreted onto the stellar surface. Assume Mira B has the same radius as Earth. The following formula may be useful. (4)

$$\frac{dU}{dt} = -\frac{GM\dot{m}}{R_B}$$

$$\Delta U = U_f - U_i - \frac{GM_A M_B}{R_B} - \left(-\frac{GM_A M_B}{d}\right), \ d \to \infty \ since \ d \gg R_B$$

$$\Delta U = -\frac{GM_A M_B}{R_B}, \ E = -\Delta U$$

$$L = -\frac{dU}{dt} = \frac{GM_B \frac{dm}{dt}}{R_B}$$

$$L = \frac{G(0.6 \, M_\odot \cdot 1.989e30 \, kg/M_\odot)}{6.378 \cdot 10^6 \, [m]}\left(0.01 \cdot 10^{-7}\left(\frac{M_\odot}{yr}\right)\left(\frac{1.989e30 \, kg/M_\odot}{365 \cdot 24 \cdot 60 \cdot 60 \, sec/yr}\right)\right)$$

$$= 7.88 \cdot 10^{26} \to 2.02 \, L_\odot$$

$$m = -26.74 - 2.5 log\left(\frac{\frac{2.02 \, L_\odot}{4\pi(300 \, [ly])^2}}{1400 \, W/m^2}\right) \to 8.9$$

insert post-test comments if you have any

This test was written by Ethan Chen (Novi '25, UMich '29) and Pradyumna Gururajan (Mason '28).

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